Preliminary work.

This book assumes that you are already familiar with a number of mathematical ideas from your mathematical studies in earlier years.

This section outlines the ideas which are of particular relevance to Unit One of the Mathematics Specialist course and for which familiarity will be assumed, or for which the brief explanation given here may be sufficient to bring your understanding of the concept up to the necessary level.

Read this "preliminary work" section and if anything is not familiar to you, and you don't understand the brief mention or explanation given here, you may need to do some further reading to bring your understanding of the concepts up to an appropriate level for this unit. (If you do understand the work but feel somewhat "rusty" with regards to applying the ideas, some of the chapters afford further opportunities for revision, as do some of the questions in the miscellaneous exercises at the end of chapters.)

☞ Chapters in this book will continue some of the topics from this preliminary work by building on the assumed familiarity with the work.

☞ The miscellaneous exercises that feature at the end of each chapter may include questions requiring an understanding of the topics briefly explained here.

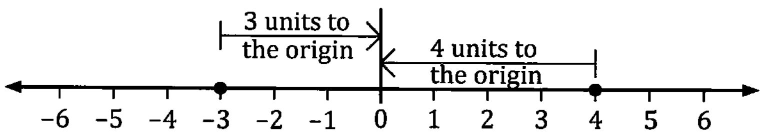
☞ Familiarity with some of the content of unit one of the *Mathematics Methods* course will be useful for this unit of *Mathematics Specialist.* It is anticipated that students will be studying that unit at the same time as this one and that the useful familiarity with content will be reached in time for its use in this unit.

- **Number.**
  The set of numbers that you are currently familiar with is called the set of **real numbers**. We use the symbol $\mathbb{R}$ for this set. $\mathbb{R}$ contains many subsets of numbers such as the whole numbers, the integers, the rational numbers, the irrational numbers etc.

- **The absolute value.**
  A concept that you may have met before is that of the absolute value of a number. This is the distance on the number line that the number is from the origin.

The absolute value of −3, written $|-3|$, is 3 because −3 is three units from the origin.

The absolute value of 4, written $|4|$, is 4 because 4 is four units from the origin.



The absolute value makes no distinction between numbers that are to the left of the origin and those that are to the right of the origin.

Thus

$|3| = 3$ $\quad |5| = 5$ $\quad |3{\cdot}1| = 3{\cdot}1$ $\quad |8{\cdot}2| = 8{\cdot}2$

$|-3| = 3$ $\quad |-5| = 5$ $\quad |-3{\cdot}1| = 3{\cdot}1$ $\quad |-8{\cdot}2| = 8{\cdot}2$

- **Trigonometry.**

  It is assumed that you are already familiar with the use of the theorem of Pythagoras and the ideas of sine, cosine and tangent to determine the unknown sides and angles in right triangles. Your answers should be given to the accuracy requested or, if none is specifically requested, to an accuracy that is appropriate for the accuracy of the given data and the nature of the situation.

  An understanding of the concept of a bearing being measured clockwise from North, eg 080°, and using compass points, eg N80°E, is also assumed.

  With $\text{sine} = \frac{\text{Opp}}{\text{Hyp}}$ $\text{cosine} = \frac{\text{Adj}}{\text{Hyp}}$ and $\text{tangent} = \frac{\text{Opp}}{\text{Adj}}$ it follows that:

  $$\boxed{\tan A = \frac{\sin A}{\cos A}}$$

  a result you may not be familiar with but that we will use in this unit.

  You may be familiar with ☞ the formula $A = \frac{1}{2} a\, b \sin C$ to determine the area of a triangle,

  ☞ the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$,

  ☞ the cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$,

  ☞ the use of the unit circle to give meaning to the sine and cosine of angles bigger than 90°,

  and ☞ expressing the trigonometric ratios of some angles as exact values,

  or you will become familiar with these concepts as your concurrent study of unit one of the *Mathematics Methods* course progresses.

  It is also assumed that you are familiar with the idea that *similar triangles* have corresponding sides in the same ratio - the underlying idea on which the trigonometrical ratios of right triangles are based.

- **Use of algebra.**

  It is assumed that you are already familiar with manipulating algebraic expressions, in particular:

  ☞ Expanding and simplifying:

For example $4(x+3) - 3(x+2)$	expands to	$4x + 12 - 3x - 6$
	which simplifies to	$x + 6$
$(x-7)(x+1)$	expands to	$x^2 + 1x - 7x - 7$
	which simplifies to	$x^2 - 6x - 7$
$(2x-7)^2$, i.e. $(2x-7)(2x-7)$	expands to	$4x^2 - 28x + 49$

  ☞ Factorising:

For example,	$21x + 7$	factorises to	$7(3x+1)$
	$x^2 - 6x - 7$	factorises to	$(x-7)(x+1)$
	$x^2 - y^2$	factorises to	$(x-y)(x+y)$.

  the last one being referred to as the *difference of two squares*.

  ☞ Solving equations: In particular linear, simultaneous and quadratic equations.

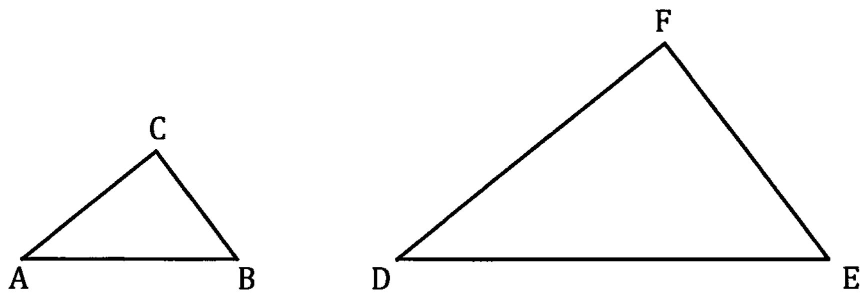
- **Similar triangles.**

Whilst it is assumed that you are familiar with the concept of two triangles being similar (as mentioned at the end of the trigonometry section of this Preliminary Work section) it is further assumed that you can apply these ideas in the formulation of a proof. Read through the following by way of a reminder of these ideas.

If two triangles are *similar* corresponding sides are in the same ratio. (One triangle is like a "photographic enlargement" of the other.)
For example, if the two triangles ABC and DEF, shown below, are similar then

$$AB : DE = AC : DF = BC : EF$$

i.e. $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$



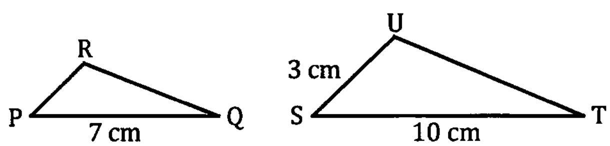
Note If two triangles are similar the corresponding angles are equal.
i.e., in the similar triangles shown above
$\angle CAB = \angle FDE$ $\angle ABC = \angle DEF$ $\angle ACB = \angle DFE$

We can use the symbol "~" to indicate that two shapes are similar.
Thus $\Delta ABC \sim \Delta DEF$
The order of the letters <u>is</u> significant and indicates corresponding sides and angles. Thus, for triangles ABC and DEF above, it would **not** be correct to say $\Delta ABC \sim \Delta FED$.

Knowing that two triangles are similar can sometimes allow us to determine the length of some of the sides. For example if we were told that for the triangles sketched below,

$$\Delta PQR \sim \Delta STU$$

then $$\frac{PQ}{ST} = \frac{PR}{SU}$$

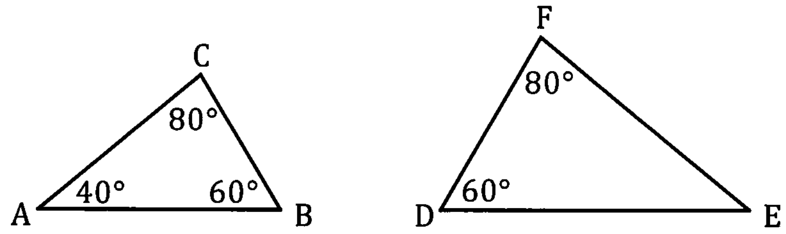


i.e. $$\frac{7}{10} = \frac{x}{3}$$ where x cm is the length of PR.

Solving gives $x = 2{\cdot}1$. Thus PR is of length 2·1 cm.

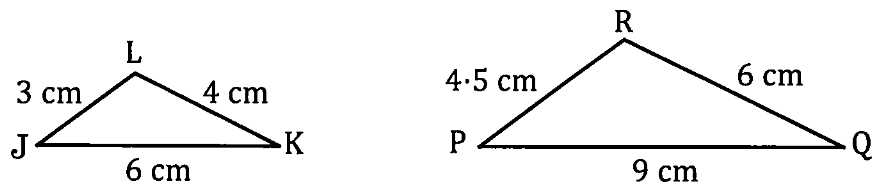
To determine whether two triangles are similar we can:

See if the three angles of one triangle are equal to the three angles of the other triangle.



ΔABC and ΔEDF are similar. Reason: Corresponding angles equal.

Or: See if the lengths of corresponding sides are in the same ratio.



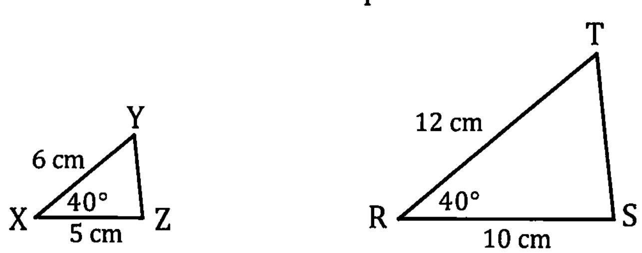
JK : PQ = 6 : 9
= 2 : 3

KL : QR = 4 : 6
= 2 : 3

JL : PR = 3 : 4·5
= 2 : 3

ΔJKL and ΔPQR are similar. Reason: Corresponding sides in same ratio.

Or: See if the lengths of two pairs of corresponding sides are in the same ratio and the angles between the sides are equal.



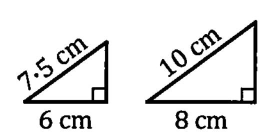
XZ : RS = 5 : 10
= 1 : 2

XY : RT = 6 : 12
= 1 : 2

The angle between XY and XZ = the angle between RT and RS.

ΔXZY and ΔRST are similar. Reason: Two pairs of corresponding sides in same ratio and the *included* angles equal.

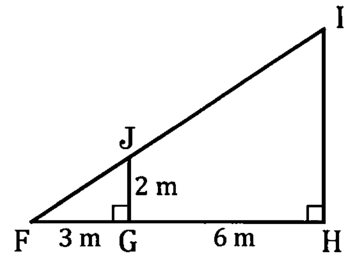
Note: If the two triangles are right angled the corresponding sides that are in the same ratio need not *include* the right angle.



Example
Given that in the diagram shown on the right

FG = 3 m, GH = 6 m, JG = 2 m.

Prove that IH is of length 6 m.



Given: Diagram as shown.
To prove: IH = 6 m.
Proof: In triangles FGJ and FHI:

	∠FGJ	=	∠FHI	(Each angle = 90°.)
	∠JFG	=	∠IFH	(Same angle.)
∴	∠FJG	=	∠FIH	(Third angle in each triangle.)
Thus	ΔFGJ	~	ΔFHI .	(Corresponding angles equal.)

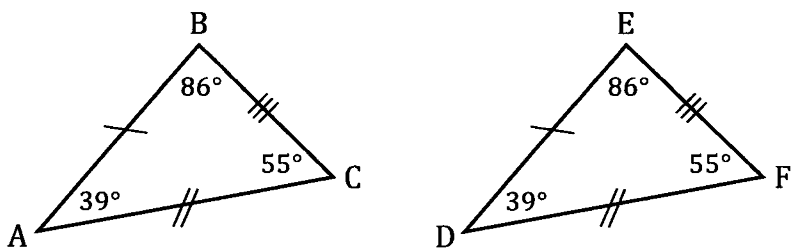
∴ $\frac{JG}{IH} = \frac{FG}{FH}$ i.e., if IH is x m in length, $\frac{2}{x} = \frac{3}{9}$

$x = 6$

Thus IH is of length 6 m, as required.

Notice how known truths are referred to in order to justify statements. Such justifications do not need to be "essays" but are instead a brief statement to clearly indicate which truth justifies the statement.

- **Congruent triangles.**
If two triangles are identical, i.e. *congruent*, the three angles and three sides of one will match the three angles and three sides of the other. For example, the triangles ABC and DEF shown below are congruent.



We write: ΔABC ≅ ΔDEF

However, we do not need to know that all six pieces of information,

A A A S S S,

"match" before being able to say that two triangles are congruent.

In fact, to prove two triangles congruent, we need only to show one of the sets of facts shown on the next page.

To prove two triangles congruent we need only show one of the following:

SSS The lengths of the three sides of one triangle are the same as the lengths of the three sides of the other triangle.

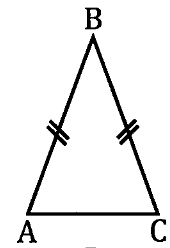
SAS The lengths of two sides of one triangle are the same as the lengths of two sides of the other triangle and the included angles are equal.

AA corresponding S.
Two angles of one triangle equal two angles of the other and one side in one triangle is the same length as the corresponding side in the other.

RHS Both triangles are right angled, the two hypotenuses are the same length and one other side from each triangle are of equal length.

Example

If we define an isosceles triangle to be a triangle that has two of its sides of the same length, prove that two of its angles must also be equal.



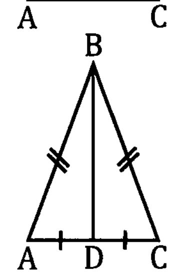
Given: ΔABC with $AB = CB$.

To Prove: $\angle BAC = \angle BCA$

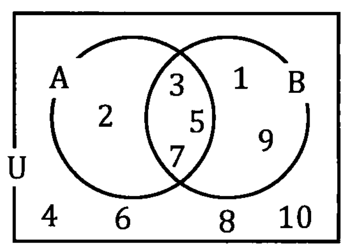
Construction: Draw the line from B to D, the mid-point of AC. (See second diagram on the right.)

Proof: In triangles ADB and CDB

$AB = CB$ (Given.)
$AD = CD$ (D is the mid-point of AC.)
$BD = BD$ (Common to both triangles.)
$\therefore$ $\Delta ADB \cong \Delta CDB$ (SSS)
Hence $\angle BAD = \angle BCD$ (Corresponding angles)
and so $\angle BAC = \angle BCA$ as required.



- **Sets and Venn diagrams.**



Some familiarity with Sets and **Venn diagrams** is assumed. The **Venn diagram** on the right shows the **universal set**, U, which contains all of the **elements** currently under consideration, and the sets A and B contained within it.

We use "curly brackets" to list a set. Thus for the sets shown $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 3, 5, 7\}$ and $B = \{1, 3, 5, 7, 9\}$.

Using the symbol $\in$ for "is a member of": $3 \in A$, $9 \in B$, $9 \notin A$.

For the union of two sets we use "$\cup$", for the intersection of two sets we use "$\cap$", for the number of elements in a set we use n(A) or |A| and for the complement of a set we use A' or $\overline{A}$. Thus for the sets A and B shown above

$A \cap B = \{3, 5, 7\}$, and $A \cup B = \{1, 2, 3, 5, 7, 9\}$,
$n(A) = 4$ and $A' = \{1, 4, 6, 8, 9, 10\}$.

Sometimes we use the Venn diagram to show the number of elements in the particular regions, rather than the elements themselves.